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THE OPTIMAL DESIGN OF STEAM AND POWER SYSTEMS IN CHEMICAL PLANTS*

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Steam and Power Systems - Optimal Design - Rational Use of Energy - Energy Conservation - Process Synthesis - Computer Aided Design.

Abstract

Minimization of investment and operating costs is proposed to obtain the optimal design of Steam and Power Systems in Chemical Plants. The mathematical model includes equipments and practical aspects not taken into account in previous work. Equipments such as the deareator, the boiler pump, heat recovery boilers, etc., are included and properties such as turbine efficiencies are calculated as dependent variables allowing the usage of correlations provided by manufacturers. The Mixed-Integer non-Linear optimization structure of the problem is replaced by an appropriate sequence of non-Linear optimization problems. A computer program provides solutions ordered in increasing cost. Therefore, capital and operating costs can be compared and the optimum can be selected from many alternatives available to the engineer. Using a few examples, the existence of alternative optimum solutions is discussed, and the structure of sub-optimum solutions is described. Finally, certain aspects of the relationship of this design problem with the design of the Heat Exchanger Network are pointed out.

I. Introduction

The Steam and Power System (SPS) of a Chemical Plant provides mechanical energy by means of backpressure and condensing turbines, and is simultaneously a heat source for the Heat Exchanger Network (HEN). As shown schematically in Figure 1, steam V_1^B produced in the Main Boiler, together with steam coming from Waste Heat Boilers V_j^R (and eventually imported steam from other plants V_j^I), flows into a system of headers having different pressures. Part of this steam is used to generate mechanical power in turbines $W_{k,j}$ whereas another portion V_j^H is used for heating process streams. The remaining part is used to meet demands of direct steam used for example as raw material or diluent in the process. To enhance the efficiency of the cycle, backpressure turbine outlets are injected into the corresponding headers, and the condensates from the heat exchangers are flashed appropriately.

As it is discussed below, the use of steam of different pressures and/or the eventual excess of it in some levels can alter HEN design goals such as the minimum utility usage proposed by Cerdá *et al.*[1]. For this reason, the design of the SPS and the HEN are interdependent. There are some cases, however, where separate designs of both systems can meet necessary optimal conditions for the combined structure. In this paper a method to find the optimal design of the SPS, using the HEN heat demand as data, is presented. The nature of the relationship between the two systems is explored, and some cases where the present method guarantees the optimality of the combined system are determined.

Many of the procedures currently used for the HEN design, which are based on the methods presented by Linhoff and Hindmarsch[2] and Cerdá and Westerberg[3], assume that the operating costs are dominant, and therefore one necessary optimal condition is minimum energy consumption. The direct association between minimum operating costs and minimum energy consumption is done on the basis that all the heat demand is satisfied using boiler steam. When the SPS is introduced as an intermediary, a new problem is defined. The heat demand is now being fulfilled by steam coming from backpressure turbine outlets or waste heat boilers, and eventually with imported steam. This new situation can produce steam excesses in low pressure levels, which can be used to meet higher HEN heat demand without changes in the boiler load. Also the heat load allocation can be critical in determining the boiler energy load. For example, Doldán *et al.*[4] showed that when

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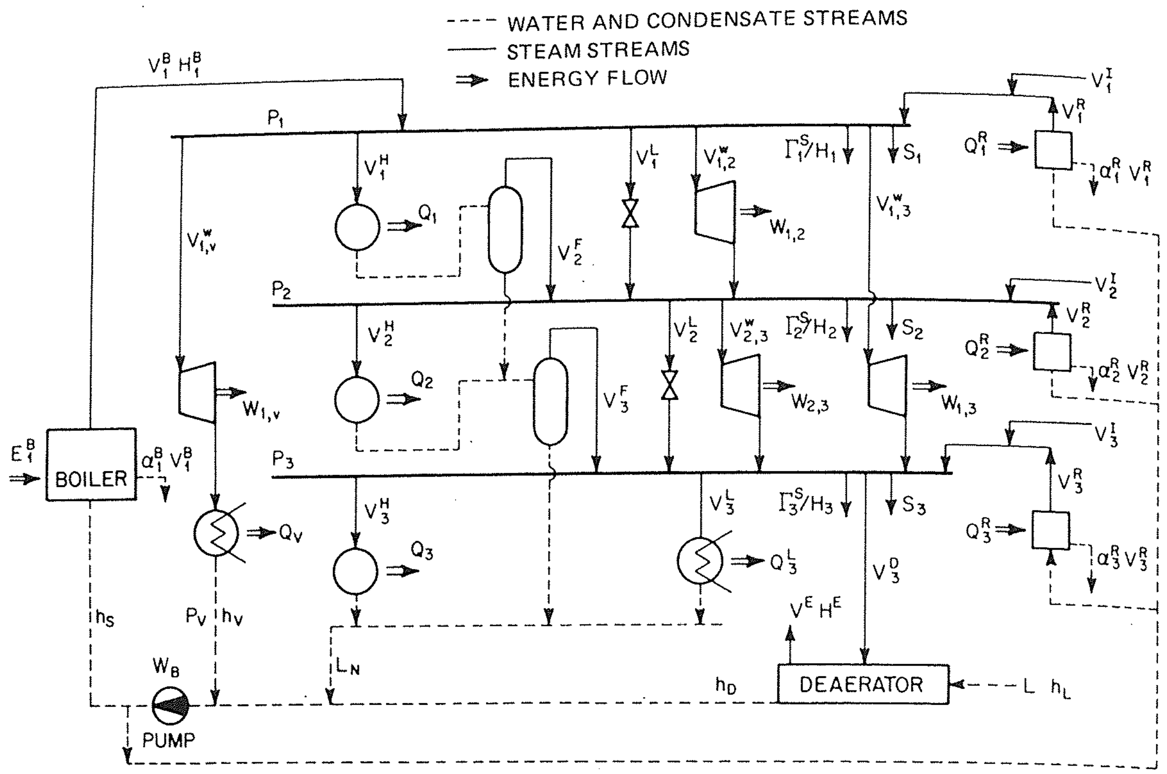

Fig. 1.– Steam and power system

the power alloted to each header remains constant, reallocation of the heat loads of the different headers can produce reductions in the boiler load. Later, Doldán *et al.*[5,6] showed that when reducing HEN heat demands, important bottleneck conditions may arise in the SPS, such that the boiler heat load does not change, being also accompanied by cooling water increments. The same type of restrictions can appear even in the case where power loads can be redistributed among the different headers, especially in the case of power dominant systems. Given the complexity of the problem the indirect heat demand is assumed in this paper to correspond to the minimum HEN utility usage. An analysis of the combined SPS-HEN system after the SPS has been designed will determine whether an increase of the HEN utility usage is possible without affecting the boiler fuel load.

Fortunately, in the case of the HEN design the minimum utility usage can be obtained without actually designing the system. Using this value, the HEN design can be approached determining which network gives the lower capital cost. A second heuristic that associates minimum capital costs with minimum number of heat exchangers, allows simplified mathematical methods to meet the design goals, as it was developed by Linhoff and Hindmarsch[2] and Cerdá and Westerberg[3]. There are, of course, other HEN design methods from which equivalent tools of analysis could be obtained. Once the SPS is designed on the basis of minimum heat demand of the HEN, the number of heat exchangers becomes a relevant parameter for the determination of the combined SPS-HEN optimum necessary conditions.

Different structures have been proposed for the SPS. Nishio *et al.*[7-10] proposed different forms of meeting the design goals. Also, Petroulas and Reklaitis[11] presented a structure, later adopted by Doldán *et al.*[4], that is similar to many industrial designs. Grossmann[12], uses the concept of "superstructure", where all the possible structures are represented simultaneously, the optimal one obtained by means of mixed-integer nonlinear programming techniques. Even in this last case the selection of the equipments and their interconnections is done beforehand. This constitutes a limitation shared by almost all the synthesis methods. In this paper, the optimal design is determined using the type of SPS structures shown in Figure 1. Except for a few additional details, this is the structure presented by Doldán *et al.*[4]. Aiming at the inclusion of certain aspects not included in earlier work, equipments such as the deaerator and the boiler pump are considered in the model. Calculation of turbine efficiencies is also included, allowing the usage of manufacturer correlations. The method allows the inclusion of other additional details needed for a complete design.

Although the inclusion of exhaustive cost evaluations of the solutions is sometimes difficult, if not inapplicable, in this paper the total cost of the system is proposed as an objective function. Once some properties of the cost function are established, the objective function and the mixed-integer nonlinear mathematical structure of the problem are replaced by an easier-to-solve mathematical scheme without any loss of rigour. Structures having minimum operating cost can be ordered in increasing values of capital costs. The minimum operating cost sub-problem is solved using a strategy that avoids the usage of mixed-integer nonlinear techniques. The overall algorithm allows independent cost evaluation according to the criteria, needs, evaluation methods and possible restrictions of each Plant or Company.

II. Description of the synthesis problem

II. 1. *Problem data*

II.1.1. *HEN heat demand*: The SPS have to meet a certain heat demand D_T which is considered fixed and corresponds to the minimum utility usage of the HEN. Using methods such as those presented by Cerdá *et al.* [1], the determination of this minimum utility usage is straightforward. Once the temperature intervals of the problem are established, the Transportation Tableau is built and solved. From this Tableau the distribution of the heat demand with respect to temperature d(T) is determined. Because of the discrete characteristic of the transportation method, this distribution has the form of the bar diagram shown in Figure 2. If the number of headers is N, only N

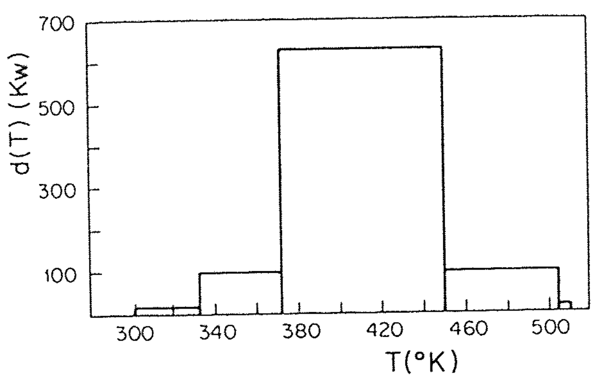

Fig. 2.– Heat exchanger heat demand distribution

different steams, each of pressure P_i and dew point T_i are available. Therefore, only N values of different heat demands can be defined. They are:

$$D_i = \int_{T_{i-1} - \Delta T}^{T_i - \Delta T} d(T) dT \qquad i = 1, \ldots, N \qquad (1)$$

where ΔT is the minimum allowed temperature difference in heat exchangers. The total fixed heat demand is then:

$$D_T = \int_0^{T_1 - \Delta T} d(T) dT = \sum_{i=1}^{N} D_i. \qquad (2)$$

In practical designs it is often found that steam of the lowest possible pressure is used to satisfy each heat demand D_i. As it will be described later a relaxation of the problem that permits the reduction in boiler loads is to allow heat demands to be satisfied by steam of higher pressures.

II.1.2. *Power demand:* Steam turbines are generally connected to electrical generators, and this electrical energy is then distributed to the energy consuming centers. In some applications, however, the power demand is specified in such a way that no electrical energy is used as an intermediary for power needs. This is the case of large compressors where efficiency in the power transmission is lowered by electrical drivers. Each of these M_D demands W_i^D is satisfied using a real turbine, called here "Dedicated Turbine". Therefore, the overall power demand can be described as the sum of the dedicated turbine demands W_i^D and the power W_E produced in turbines connected to electrical generators. Thus:

$$W_T = W_E + \sum_{i=1}^{M_D} W_i^D. \qquad (3)$$

The optimal design procedure has to assign dedicated turbines to a pair of headers. At the same time, the power distribution that will produce W_E can be assigned to any pair of headers and partitioned conveniently.

II.1.3. *Direct process steam demands:* Steam injection in the process is often present in Chemical Plants. For this reason it cannot be recovered, as is the case of the steam used to meet HEN heat demands. This demand is specified in two forms. In certain cases the energy content of the demand is important and the specification comes as Energy Flow Demand Γ_i^G. Each of these M_G values are specified to be provided at a value of pressure higher than a specified limit P_i^G, but as close as possible to it. In other cases, steam flow rates are much more important than their energy content, and they are specified as such. For these M_F values of fixed flow rate steam demand F_i, a lower limit on the supply pressure P_i^F is also specified. By selecting the lowest possible pressure for the steam that meets this demand the amount of energy associated with these streams is minimized. This also helps to minimize the boiler energy load by allowing turbine outlets to be used for these purposes. Once the header pressures are known, the process steam demands of each header are immediately established through the following relations:

$$\Gamma_j^S = \sum_{i=1}^{M_G} a_{i,j}^G \Gamma_i^G \qquad (4)$$

$$S_j = \sum_{i=1}^{M_F} a_{i,j}^F F_i, \qquad (5)$$

where $S_j + \Gamma_j^S/H_j$ is the flow rate of process steam provided by the header j. The binary variables $a_{i,j}^G$ and $a_{i,j}^F$ are used to make the supply-demand allocation. For each value of j one and only one $a_{i,j}$ is different from zero, as happens in all assignment problems.

II. 1. 4. *Steam raised in heat recovery boilers and imported steam:* There are cases in which there is steam imported from other plants or parts of a Chemical Complex, called Imported Steam V_j^I ($j = 1, \ldots, M_I$) and available heat Q_i^R throughout the process, which is used to generate steam of flow rate V_i^R. For each Heat Recovery Boiler there is a small blowdown $\alpha_j^R V_j^R$. The properties of these steams are problem data. If $a_{i,j}^R$ is a binary variable used to assign heat recovery boiler i to header j, and similarly $a_{i,j}^I$ allocates the imported steam V_i^I to header j, then

$$V_j^R = \sum_{i=1}^{M_R} a_{i,j}^R \frac{Q_i^R \epsilon_i^R}{(H_i^R - h_S) + \alpha_i^R (h_i^R - h_S)} \qquad (6)$$

$$V_j^I = \sum_{i=1}^{M_I} a_{i,j}^I V_i^I \qquad (7)$$

where h_S the boiler feed water enthalpy, $\alpha_i^R V_i^R$ is the amount of blowdown, h_i^R its enthalpy, and H_i^R the enthalpy of the steam produced by the heat recovery boiler. Finally ϵ_i^R represents the boiler efficiency, which is considered constant and data. It should be mentioned that instead of assigning heat recovery boilers to headers by matching their pressures, the problem can be generalized to include the selection of the temperature and pressure of the steam raised by the heat Q_i^R.

There are cases in which all this mass and energy supply is substantially higher than the power and heat demands. In such cases, the complex structure given in Figure 1, built to maximize the Rankine Cycle efficiency, is no longer economically justified. Nor is sometimes the existence of the main boiler itself. This situation is easily recognizable by the design engineer and since it requires a completely different analysis, it will not be studied here.

II. 1. 5. *Main boiler steam properties:* In this paper it is assumed that the properties of the main boiler steam V_1^B, are data.

Increasing boiler pressures can only increase the efficiency of steam turbines. This change in efficiency is not significant. However, higher boiler pressures sharply increase steam and energy losses. On the other hand, if the boiler steam temperature increases, there is no apparent effect on the efficiency of the turbines, at least on the basis of the correlations here used. Thus, any temperature increment raise energy losses, with apparently no other effect. Therefore, not existing a clear benefit from raising the temperature and pressure of the Main Boiler, these properties are chosen to be as small as possible. The lower limit is established by avoiding for example the appearance of two phase flow at turbine outlets.

If it is so desired, these variables can be included as problem variables without affecting the solution algorithm, provided the appropiate restrictions are also added. Additionally, it has to be mentioned that temperature increments in boiler steam allow meeting heat demands of higher level. These demands are usually satisfied with fire heaters. The new assignment problem that arises from these two competing energy suppliers is not considered in this paper.

II. 2. *Problem objective function*

As was mentioned earlier the optimal solution is obtained by minimizing the overall cost of the system. Later it will be explained how the mathematical structure of the problem is replaced by a sequence of sub-problems for which only operating costs are minimized. This operating cost is proportional to the energy boiler load E_1^B defined by:

$$E_1^B = \frac{V_1^B}{\epsilon_1^B} \left[(H_1^B - h_S) + \alpha_1^B (h_1^{(s)} - h_S) \right] \quad . \qquad (8)$$

The main boiler blowdown is $\alpha_1^B V_1^B$, the enthalpy of the steam produced is H_1^B and the boiler efficiency is ϵ_1^B. They are all data. Other expenses, such as the maintenance cost, can be easily included in the problem as structure dependent capital costs or, as is done in some cases, as functions of the operating cost.

II. 3. *Problem feasible region*

The feasible region is defined by the following restriction equations

II. 3. 1. *Fullfillment of process heat load:* By assigning a heat load Q_j to each steam header the total heat demand is fullfilled. Thus

$$\sum_{j=1}^{N} Q_j = D_T + D_L. \qquad (9)$$

Note that Q_j is not necessarily equal to D_j. Aside from the fixed heat demand D_T, given by the HEN minimum utility usage, the system itself consumes a certain amount of energy D_L needed for its own functioning. This additional load has different origins, one of which is the heat demand of the water treatment plant. This type of demand will be always dependent on the system overall energy load.

Without loss of generality it is here assumed that D_L is proportional to the water make-up L:

$$D_L = \alpha_L L \qquad (10)$$

but other formulas can be used. The temperature level of D_L is assumed to be the same as that of D_N.

Other restrictions reflecting thermodynamic constraints are:

$$\sum_{j=1}^{k} Q_j \geq \sum_{j=1}^{k} D_j \qquad k = 1, \ldots, (N-1) \qquad (11)$$

$$0 \leq Q_j \leq \sum_{k=j}^{N} D_k \qquad j = 1, \ldots, N. \qquad (12)$$

II.3.2. *Overall mass balance:* This balance is

$$\sum_{j=1}^{N} V_j^I + L = V^E + \sum_{j=1}^{N} [S_j + \Gamma_j^S/H_j + V_j^R \alpha_j^R] + \alpha_1^B V_1^B \qquad (13)$$

The deaerator steam outlet V^E is a value that depends on certain characteristics of the design of the equipment. In each case, manufacturer's data have to be consulted. For demonstrative purposes, and without loss of generality, the following linear relationship is adopted:

$$V^E = \alpha_D V_N^D. \qquad (14)$$

As will be seen later, the use of more complicated formulas can be included without affecting the ability of the algorithm to solve the problem.

II.3.3. *Mass-balance for each header:*

$$V_j^B + V_j^I + V_j^R + \sum_{k=1}^{j-1} V_{kj}^w + V_{j-1}^L + V_j^F = V_{j,v}^w + \sum_{k=j+1}^{N} V_{j,k}^w + \frac{Q_j}{(H_j - h_j^{(s)})} + V_j^L + \Gamma_j^S/H_j + S_j + V_j^D \qquad (15)$$

where $V_{k,j}^w$ is the steam flow corresponding to the backpressure turbine $W_{k,j}$ and $V_{j,v}^w$ is a condensing turbine flow rate. Condensing turbines are here considered as consumers of steam of high pressure only, therefore $W_{j,v} = 0$ for all $j \neq 1$. Similarly, only one Main Boiler is considered, so that V_j^B has meaning only for $j = 1$, whereas V_j^D, which is the steam destinated to cover deaerator duties, is supplied by the last header N, being zero for all $j \neq N$. Some of these are simplifying assumptions that can be relaxed. Others, such as the steam used in the deareator are guided by common sense.

The amount of steam recovered in each flash V_j^F is given by a linear relationship in Q_k.

$$\left.\begin{aligned} V_2^F &= \alpha_2 \frac{Q_1}{H_1 - h_1^{(s)}} \\ V_j^F &= \alpha_j \left\{ \frac{Q_{j-1}}{H_{j-1} - h_{j-1}^{(s)}} + \sum_{k=1}^{j-2} \frac{Q_k}{H_k - h_k^{(s)}} \prod_{q=k+1}^{j-1} (1 - \alpha_q) \right\}; \quad j \geq 3 \end{aligned}\right\} \qquad (16)$$

where H_j is the enthalpy of the steam in header j, $h_j^{(s)}$ is the enthalpy of the corresponding condensate and α_q is a flash splitting factor:

$$\alpha_q = \frac{h_{q-1}^{(s)} - h_q^{(s)}}{H_q^{(s)} - h_q^{(s)}}. \qquad (17)$$

Finally, the steam needed to cover the deaerator duty V_N^D is obtained combining mass and energy balances for this equipment:

$$V_N^D = \frac{L(h_D - h_L)}{[H_N - (1 - \alpha_D) h_D - \alpha_D H^E]}. \qquad (18)$$

The enthalpies H^E and h_D correspond to the deareator outlet steam V^E and its outlet water, respectively.

II.3.4. *Nonisentropic steam turbine expansion:* In each turbine, the relationship between steam flow rate and power produced is given by

$$W_{k,j} = V_{k,j}^w \eta_{k,j} [H_k - H_{k,j}^{es}] \qquad (19)$$

where $H_{k,j}^{es}$ is the enthalpy corresponding to the isentropic expansion of steam having pressure P_k and enthalpy H_k to pressure P_j.

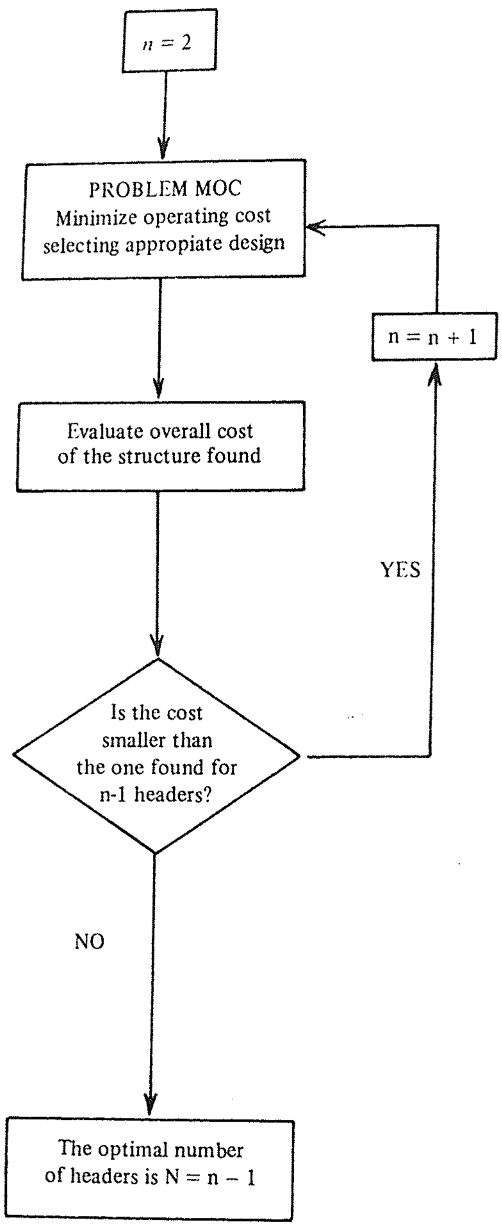

Fig. 4.– Problem solution algorithm.

the cycle can be used to identify these situations, stopping useless computations.

The definition of the overall cycle efficiency given by (29) does not take into account losses originated in changes of turbine efficiencies, and therefore it cannot be used as a mathematical alternative objective function of the problem. However, as will be shown in the examples these differences can be neglected in practice. Also, when header pressures change, the existence of direct process steam demands of fixed flow rates F_i can also induce changes in the efficiency of the cycle. This effect is not taken into account in the expression given in (29) either, and therefore, when conclusions are made based on cycle efficiency, carefull attention is needed to analyze the influence of these parameters.

III. 2. *The minimum operating cost problem (MOC)*

Even though the number of headers is fixed in the case of the MOC problem, it still has binary variables. The lack of knowledge of the steam header pressures P_i make it impossible to make the supply-demand assignment established in[4, 7]. Additionally, the integrals given by Eq. (1) remain undetermined. If mixed-integer programming can take care of the binary variables $(a^F_{i,j}, a^G_{i,j}, a^R_{i,j}, a^I_{i,j})$, it is difficult to handle restrictions containing integral expressions.

All these difficulties are eliminated if by some means, the header pressures are known. This idea suggests the definition of the following Minimum Operating Cost with Fixed Pressures Problem (MOCFP).

For each set of Steam Header Pressures the following mapping is proposed:

$$f^N(P_1, \ldots, P_N) =$$

$$= \text{Min } \{V^B_1 (H^B_1 - h_S) + \alpha^B_1 V^B_1 (h^{(s)}_1 - h_S)\} \quad (30)$$

where the minimum of the function in the right hand side have to be in the feasible region of the problem. Since this expression represents the energy input to the boiler, a unique value is obtained for each set $\{P_1, \ldots, P_N\}$. Thus, using this mapping, the minimum of the MOC problem is obtained by finding the minimum of f^N, conditioned only by the monotonicity of the header pressures given by (28).

The MOC problem has been reduced to solve the MOCFP problem. For each set of pressures the binary variables $(a^F_{i,j}, a^G_{i,j}, a^R_{i,j}, a^I_{i,j})$ are calculated and the values of D_i evaluated. As stated, the binary variables for the assignment of direct process steam demands to headers are obtained by choosing the lowest pressure possible. In the same way, to make maximum use of the steam raised in heat recovery boilers and the imported steam they are assigned to headers with the highest pressure possible. With this done, the MOCFP problem constitutes a nonlinear constrained optimization problem. In the case of $N = 2$, f^2 is a function of one variable, namely P_2 (recall that P_1 and H^B_1 are data). The Fibonaci Search was chosen to find the optimal solution in the interval $(P_1, P_{N_{Min}})$. For $N \geqslant 3$, the Pattern Search algorithm (Fould[14]) is used. To make sure that the search is done inside the feasible region of pressures the algorithm was slightly modified to avoid violations of the inequalities given by (28).

III. 3. *Iterative solution of problem MOCFP*

The MOCFP problem does not contain binary variables, but still many restriction equations are nonlinear, the objective function being bilinear in enthalpy and flow rates.

The following representation of the problem is adequate to introduce the iterative approach used in this paper. Define the vectors $\underline{x}$ and $\underline{H}$ as follows:

$$\underline{x} = (V_1^B, W_{1,\nu}, W_{k,j}, V_{1,\nu}^w, V_{k,j}^w, Q_j, V_j^L, V_j^F, L, L_N) \quad (31)$$

$$\underline{H} = (H_j, V_N^D, V^E, \eta_{k,j}, D_L, W_B, h_S, V_j^R). \quad (32)$$

In this notation the MOCFP problem is written as:

Min.

$$f^N = \underline{c}^T (\underline{H})\underline{x} \quad (33)$$

$$\underline{\underline{A}}\underline{x} - \underline{b} \geqslant 0 : [\textit{Eq. } (11) \textit{ and } (12)] \quad (34)$$

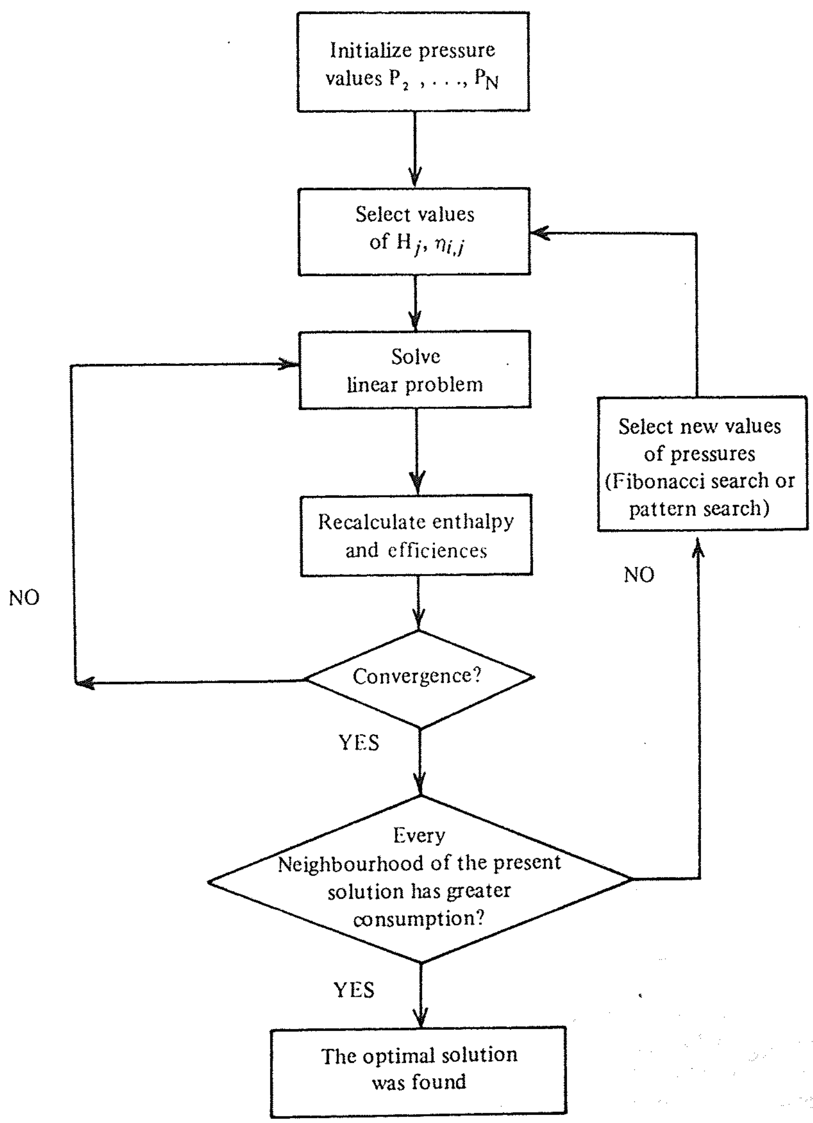

Fig. 5.– Solution procedure for problem MOC.

$$\underline{g}(\underline{x}, \underline{H}) = 0: [Eq. (9), (13), (15\text{-}16), (19) \text{ and } (24\text{-}25)] \quad (35)$$

$$\underline{H} - \underline{f}(\underline{x}, \underline{H}) = 0: [Eq. (6), (10), (14), (18), (20\text{-}23) \text{ and } (26)] \quad (36)$$

The linear problem that results from considering $\underline{H}$ constant in Eqs. (33), (34) and (35) is solved using the standard Simplex algorithm. The system of equations represented by (36) is used to recalculate new values of $\underline{H}$, as it is shown in Figure 5. This iterative procedure defines a sequence of solutions which on convergence, gives a solution of the MOCFP problem. This approach is similar to the inside-out algorithm which solves rigorous models in an outerloop of the optimization problem. The convergence to optimal conditions was investigated by Biegler *et al.*[15]. The conditions under which the sequence $\{x^k, \underline{H}^{(k)}\}$ exists and converge were not investigated in detail. Note however that this study includes a definition of a transformation in a proper metric space, and a search for conditions under which the transformation is a contraction. Such analysis is beyond the scope of this paper.

It was mentioned before that the definition given for the variable heat and power demands D_L, V_N^D, W_B respectively, as well as the expressión (20) used to calculate turbine efficiency can be replaced without altering the solution strategy here presented. All those expressions are included in (36), and therefore do not affect the existence of the linear optimization problem defined for each iteration.

Table 1. – *Heat demand distribution*

Temperature (°K)	d_s (T) (Kw/°K)
301-333	5.0
333-373	93.5
373-450	623.5
450-503	93.5
503-510	5.0
401	20000 Kw*

* Condensing stream

IV. Results

Three application examples were chosen to discuss relevant aspects of the problem. The problem data is given in Tables 1 and 2. The solutions of the corresponding MOC problems are shown in Table 3 and figures 8, 9 and 10. Normally around 15 iterations are needed to obtain convergence of the MOCFP problem with errors less than 0.0001% in the iteration variables. Such low convergence values were needed to obtain good values of f^N for the purpose of comparing sub-optimal solutions, as is shown later. The initial values of header enthalpies and efficiencies were the convergence values corresponding to the last set of pressures. For the first set of pressures saturated steam enthalpies were taken as starting values. In the case of efficiencies a value of one was chosen. No appreciable difference in number of iterations was noted between one choice and another. The number of evaluations of the Pattern Search algorithm are strongly dependent on the starting set of pressure values. A practice that gave excellent results in to make a preliminary evaluation of selected isolated points, picking the starting set on the basis of this survey. Being this type of exploratory movements strongly dependent on the engineer's judgement they cannot be programmed in a systematic way.

IV. 1. *Changes in the number of headers N*

Consider first Example 1. In the case of two headers (N = 2), the SPS includes a condensing turbine. For Power Dominant systems the maximum value of the Rankine cycle efficiency given by (29) is smaller than unity. As discussed above, the addition of one header increases the system efficiency. This happens in the case of Example 1 when passing from two to three headers. A considerable decrease in operating cost is achieved. Since operating costs are proportional to the boiler load, the large reduction of boiler load achieved (about 5%) suggests that at least two solutions (N = 2 and N = 3) should be compared in total cost. Since for N = 3 the efficiency is maximum, any increase in headers do not produce energy savings. Additionally, as it is discussed later, if variations in energy boiler load originated in changes of turbine efficiency are neglected, many alternative optimum solutions for N = 3 may exist.

In other cases, when power demands are larger than heat demands, the cycle efficiency meets its maximum value for very high and economically unattractive values of N, or eventually never reaches it. Sometimes, the limitation is not in the relatively high ratio of power to heat demand, but relies in the shape of the distribution of heat demand d (T), making it very difficult to allocate backpressure turbine outlet steam to satisfy heat demand.

It should not be surprising that in practical applications, structures having more than three headers are very rare. Savings in operating costs are normally high when passing from two to three headers, decreasing substantially from then on. As explained, new headers contribute to decrease the value of D_1, which is normally small compared to D_N, producing energy savings that are small compared to the increments in capital costs.

Table 2.– *Examples data*

For all the examples:		
$W_T = 7240$ Kw	$W_0 = 298.3$ Kw	$m = 0.0873$
$P_1 = 3$ MPa	$P_N^{Min} = 0.11$ MPa	$P_\nu = 0.01$ MPa
$h_L = 1062.08$ KJ/Kg	$h_D = 4199.4$ KJ/Kg	$T_1^B = 530$ °K
$H^E = 26760.5$ KJ/Kg	$\alpha_1^B = 0.01$	$\alpha_D = 0.05$
$\alpha_L = 10.62$ KJ/Kg	$\alpha_W = 1740$ (Pa m³/Kg)	
$\eta_0 = 0.56 - 0.0145$ P (MPa)		
Heat demands:		
* Example 1 and 3:	Table 1	
* Example 2:	Condensing streams	
	$D_1 = 20000$ Kw (@ 510 °K)	
	$D_2 = 56900$ Kw (@ 270 °K)	
Steam data		
* Example 1 and 2:	No supply or demand	
* Example 3:		
$F_1 = 37$ Ton/h	$F_2 = 2.0$ Ton/h	$F_3 = 5.0$ Ton/h
$P_1^F = 3.0$ MPa	$P_2^F = 2.0$ MPa	$P_3^F = 0.4$ MPa
$V_1^I = 28$ Ton/h	$P_1^I = 3.1$ MPa	$T_1^I = 523$ °K
$Q_1^R = 3000$ Kw	$P_1^R = 2.5$ MPa	$T_1^R = 513$ °K
$\alpha_1^R = 0.01$	$\Gamma_1^G = 2700$ Kw	$P_1^G = 2.0$ MPa

IV. 2. *Steam flow of letdown between headers*

Any steam letdown inmediately affects the boiler energy load. Letdowns between headers represent power losses, and the optimal structure avoids having such flows. This is accomplished increasing high pressure heat loads Q_i as discussed by Doldán *et al.*[4].

When an excess of steam is present in high pressure headers, originated for example by an excess of imported steam, the optimal solution can include steam letdowns to cover heat and/or direct process steam demands of lower pressure levels. Increments of the total power produced W_T can be planned knowing that turbines can be installed instead of these letdowns without affecting the overall energy load of the system.

IV. 3. *Boiler steam flow rate as objective function*

Because of the presence of h_S in the objective function the strict equivalence between minimizing V_1^B and E_1^B, as it was proposed in previous work, is no longer possible. Neglecting small variations originated in changes of turbine efficiency and variable loads (V_N^D, D_L), the enthalpy of the boiler feed water h_s remains indifferent to structural changes, as long as $W_{1,\nu}$ is constant or zero.

In Example 1, when N = 2, values of V_1^B lower than those of the optimum solution were obtained for slightly lower values of pressure P_2. For example, if $P_2 = 0.3415$ MPa, $V_1^B = 141.7$ Ton/h is obtained as solution of the MOC problem. This value is lower than the optimum Boiler Flow Rate. However in this case, f^2 increases 3.4%.

The use of V_1^B *as an alternative objective function is not recommended for the optimum design of the SPS, except for estimation purposes when short cut algorithms are used.*

Additionally, it should be mentioned that the use of V_1^B as objective function does not introduce any particular improvement in the iterative procedure used to solve the MOCFP problem.

IV. 4. *Effect of the heat demand distribution* d (T)

Example 2 differs from Example 1 only in the heat demand distribution d(T) and was included to show how changes in this distribution affect the value of the optimal solution. In this case, backpressure turbines can be used to meet all the power demand, their outlet being used to cover heat demand D_2. The optimal value of P_2 coincides with the minimum value P_2^{Min}. This is explained by the fact that the steam needed to cover deaerator duties V_2^N decreases with header enthalpy H_2. Maximum efficiency is achieved for N = 2, and this is the problem optimum. Note that no capital cost evaluations are needed when this type of results are obtained.

IV. 5. *Process steam demands and heat recovery boilers*

Example 3 is included to show how the method handles the presence of process steam demands and additional energy inputs. Not surprisingly, optimal pressures coincide with minimum values P_2^F and P_3^F specified for the steam injection demands F_2 and F_3. The energy flow associated with the steam S_2 and S_3 increases with steam header pressure. In the case of Example 3, any decrease of pressure P_3 below P_3^F will reassign steam demand F_3 to header 2, increasing the energy load of the system. Similar reassignment happens if P_2 falls below the optimum value P_2^F. Thus, the presence of steam injection demand of fixed flow rate make the minimum delivery pressures P_j^F act as "attractors" for the optimum header pressures. If M_F is higher than N, or there are limitations produced by the heat demand distribution d(T), the optimum header pressures may not coincide with these attractors. Knowing this attracting behaviour, preliminary evaluations using these pressure values may be of great help in the search algorithm.

When process steam demands are specified to be provided at fixed energy flow, the values of P_j^G do not have the same attracting effect, since now the

Table 3.– *Optimum solutions of the examples*

Example 1 (N = 2)	f^2 = 88948.38 Kw	V_1^B = 144.04 Ton/h
$Q_1 = D_1$ = 20234.0 Kw	$Q_2 = D_2$ = 56670.0 Kw	$W_{1,2}$ = 6177.2 Kw
$W_{1,\nu}$ = 1132.4 Kw	P_2 = 0.656 MPa	
Example 1 (N = 3)	f^3 = 84530.1 Kw	V_1^B = 132.40 Ton/h
Q_1 = 17979.0 Kw	Q_2 = 16627.8 Kw	$W_{1,\nu} = W_{2,3}$ = 0.
D_1 = 13648.5 Kw	D_2 = 20958.5 Kw	$W_{1,2}$ = 1108.3 Kw
$W_{1,3}$ = 6195.7 Kw	P_2 = 0.85 MPa	P_3 = 0.355 MPa
Example 2 (N = 2)	f^2 = 84522.0 Kw	V_1^B = 147.3 Ton/h
Q_1 = 34417.4 Kw	Q_2 = 42486.3 Kw	$W_{1,2}$ = 7299.9 Kw
$W_{1,\nu}$ = 0	P_2 = 0.11 MPa	
Example 3 (N = 2)	f^2 = 100345.5 Kw	V_1^B = 160.10 Ton/h
$Q_1 = D_1$ = 21082 Kw	Q_2 = 55880 Kw	$W_{1,2}$ = 6368.7 Kw
$W_{1,\nu}$ = 948.6 Kw	P_2 = 0.634 MPa	$S_1 = F_1 + F_2$
$S_2 = F_3$	$\Gamma_1^S = \Gamma_1^G$	$a_{1,1}^I = a_{1,2}^R = 1$
Example 3 (N = 3)	f^3 = 96428.3 Kw	V_1^B = 150.65 Ton/h
Q_1 = 6131.3 Kw	D_1 = 2601.0 Kw	Q_2 = 25804.9 Kw
D_2 = 29336.1 Kw	$W_{1,2}$ = 660.4 Kw	$W_{1,\nu} = W_{2,3}$ = 0
$W_{1,3}$ = 6652.4 Kw	P_2 = 2.0 MPa	P_3 = 0.4 MPa
$a_{1,1}^I$ = 1	$S_1 = F_1$	$S_2 = F_2$
$S_3 = F_3$	$a_{1,2}^R$ = 1	$\Gamma_2^S = \Gamma_1^G$
	Alternative solution	
Q_1 = 3695.5 Kw	Q_2 = 28240.7 Kw	$W_{1,2}$ = 2151.3 Kw
$W_{1,\nu} = W_{1,3}$ = 0	$W_{2,3}$ = 5161.5 Kw	P_2 = 2.0 MPa
P_3 = 0.4 MPa		
In all cases $V_i^L = 0$		

energy output associated with process steam injection is constant. However, in some cases the backpressure turbine outlet, which would otherwise be in excess in lower pressure headers, can be used to cover the direct process steam demand. In Example 3, if $P_2 < P_1^G$ the steam demand Γ_1^G is satisfied with steam coming from the first header. In this case, little variations in boiler load f^3 are observed because the system can rearrange the heating (Q_i) and Power ($W_{i,j}$) loads accordingly. This is not always possible, especially in those cases where cycle efficiency is not maximum.

Similar comments can be made to describe the effect of the presence of imported steam V_j^I and/or steam coming from Heat Recovery Boiler V_j^R. These steams act as substitutes of the Main Boiler Steam, and as such can be used in backpressure turbines. If the pressure of the header where this steam is injected is not maximized, an excess of steam can appear in lower pressure headers. On the other hand, the pressure of these steams is in general different from the optimum pressure of the headers. This introduce some work losses, such as the one seen in the optimal solution of Problem 3 where V_1^R has to feed a header having a pressure 0.5 MPa lower. The inclusion of backpressure turbines to make use of all this lost work is convenient. The policy adopted in this paper was to assign these steams to headers having the highest pressure possible, allowing its utilization in turbines before any other demand is satisfied. When a turbine is included before any injection in headers, the outlet steam can now feed any header.

IV. 6. *Alternative optima and dedicated turbines*

In all Process Synthesis problems, the existence of alternative optima or sub-optimal solutions is of much interest. The posibility of obtaining many different structures of the same or similar cost to compare, is a situation very appealing to Process Design Engineers.

Alternative optima having different power loads $W_{i,j}$ are of great interest in the case of the SPS design problem. Cost, safety and operability criteria, among others, can be used to select the best design with greater degree of freedom. From the point of view of the assignment of dedicated turbines these alternative optima may allow the possibility of having restriction (27) satisfied, without additional computations. Since the alternative solutions form a set in which the power allocation between headers changes in a continuous manner inside some interval, it is a simple matter for a process engineer to check if this power allocation can be made for some subset of these solutions.

Alternative optima are present when the system cycle efficiency reaches the maximum value. In the absence of condensing turbines, the pressure of the intermediate steam headers ($P_2, \ldots, P_{N-1}$) can vary freely, giving different sets of optimum $W_{k,j}$ values, without affecting the system energy load. This can be easily seen if an energy balance enclosing all steam headers is done neglecting all variations originated in turbine efficiency changes. In turn, if the system does not have maximum cycle efficiency, all changes in intermediate steam header pressures affect the value of the power produced in condensing turbines. This inmediately lowers the efficiency of the cycle. The same analysis can be done when process steam demand S_j is present. As shown before, increments in header pressure increase the energy output, and therefore no alternative optima can exist.

When N = 2, no alternative optima are possible, since no intermediate steam headers are present in this case. For the case of three or more headers, once $W_{1,P} = 0$ is obtained, the values of the intermediate pressures can vary in a certain range. For the Example 1, relevant values of alternative solutions are shown in Figure 6. For P_2 greater than 1.45 MPa, the alternative solution having $W_{1,3}$ different from zero takes off, giving slightly higher boiler loads. The heat load distribution is very similar for both cases, and only the corresponding to the solution having $W_{1,3} = 0$ is shown.

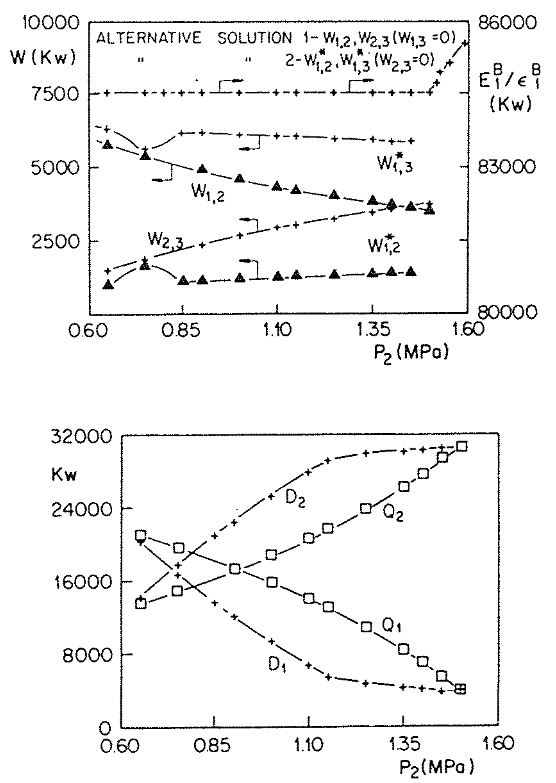

Fig. 6. – Selected values of alternative solutions of example 1. 6.*a:* Boiler load and power allocated to turbines; 6.*b:* Heat load and heat demand distribution.

Strictly speaking, these alternative optima do not exist because the turbine efficiencies vary when inlet pressure and power alloted to each turbine change from structure to structure. However, the effect of these variations on the boiler load is less than 0.016% for the plateau shown in Figure 6. For practical purposes this can be ignored. This is the reason why this sub-optimal structures are still called "Alternative Optima".

IV. 7. *Sub-optimal structures*

During all the steps of the Fibonacci Search or the Pattern Search, many solutions of the MOC problem are found. Therefore, one can obtain solutions of the problem that have values of the objective function slightly above the optimum. For many reasons, these solutions can be of practical interest. Figure 7 shows the importance of such studies. In this figure, the sensitivity of the boiler load E_1^B to changes of P_3 is shown for the case of Example 1. Below the optimum value, a sharp increase of boiler load is observed basically produced by an increase of $W_{1,\nu}$, whereas for values of P_3 over the optimum, $W_{1,\nu}$ is zero in a certain interval. Subtle changes in boiler load are observed and they are produced by changes in the variable demands V^D, W_B y D_L. As happens in the case of the alternative optima, for each value of P_3 in these sub-optimal solutions, different values of P_2 share the same boiler load. For example, for P_3 = 0.455 MPa and P_2 = 1 MPa, E_1^B = 84.554 Kw

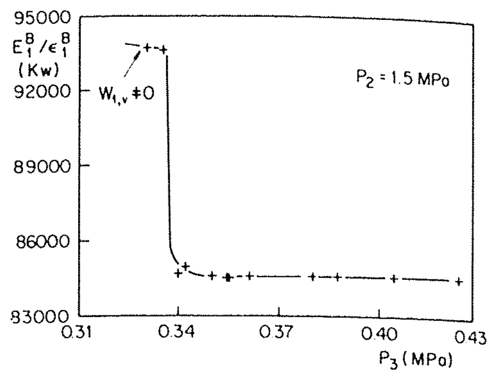

Fig. 7.- Boiler load for subóptimum solutions of example 1.

Table 4.- *Sub-optimum solutions.*

(Example 3, N' = 3)

P_2 (MPa)	P_3 (MPa)	f^3 (Kw)	$W_{1,2}$ (Kw)	$W_{1,3}$ (Kw)	$W_{2,3}$ (Kw)
2.	0.4	96428.3	660.4	6652.4	-
2.	0.4	96433.3	2151.3	-	5161.5
2.01	0.4	96428.6	664.3	6648.5	-
2.01	0.4	96433.7	2136.0	-	5176.7
1.99	0.4	96455.5	659.3	6653.5	-
1.99	0.4	96765.9	2129.0	-	5184.0
2.	0.41	96432.2	621.2	6691.7	-
2.	0.41	96437.3	2139.6	-	5173.3
2.	0.45	96447.9	497.4	6815.9	
2.	0.45	96452.9	2120.0	-	5193.4
1.95	0.4	96455.0	643.6	6669.2	
1.95	0.4	96466.2	2231.7	-	5081.1
1.8	0.4	96453.3	584.1	6728.7	-
1.8	0.4	96466.7	2481.8	-	4831.0
1.	0.4	96464.3	2282.4	5030.4	-
1.	0.4	96470.7	4433.2	-	2879.6
0.9	0.45	96477.6	1849.3	5464.0	-
0.9	0.45	96492.3	5026.8	-	2286.6

is obtained. This value barely differs from the optimum and two different power load distributions share the same boiler load. One of them has $W_{1,2}$ = 818 Kw and $W_{1,3}$ = 6486.5 Kw whereas the other has $W_{1,2}$ = 4919.5 Kw and $W_{2,3}$ = 2385.0 Kw.

For the case of the Example 3, selected values of sub-optimal structures are shown in Table 4. Note that the pair P_2 = 1.99 MPa, P_3 = 0.4 MPa is included as sub-optimal solution even though F_2 is now being satisfied by steam supplied by header 1. The systems find ways to compensate for this increase in energy flow associated with direct process steam. In the case where P_3 = 0.39 MPa the Main Boiler demand raises to 98554 Kw, indicating that

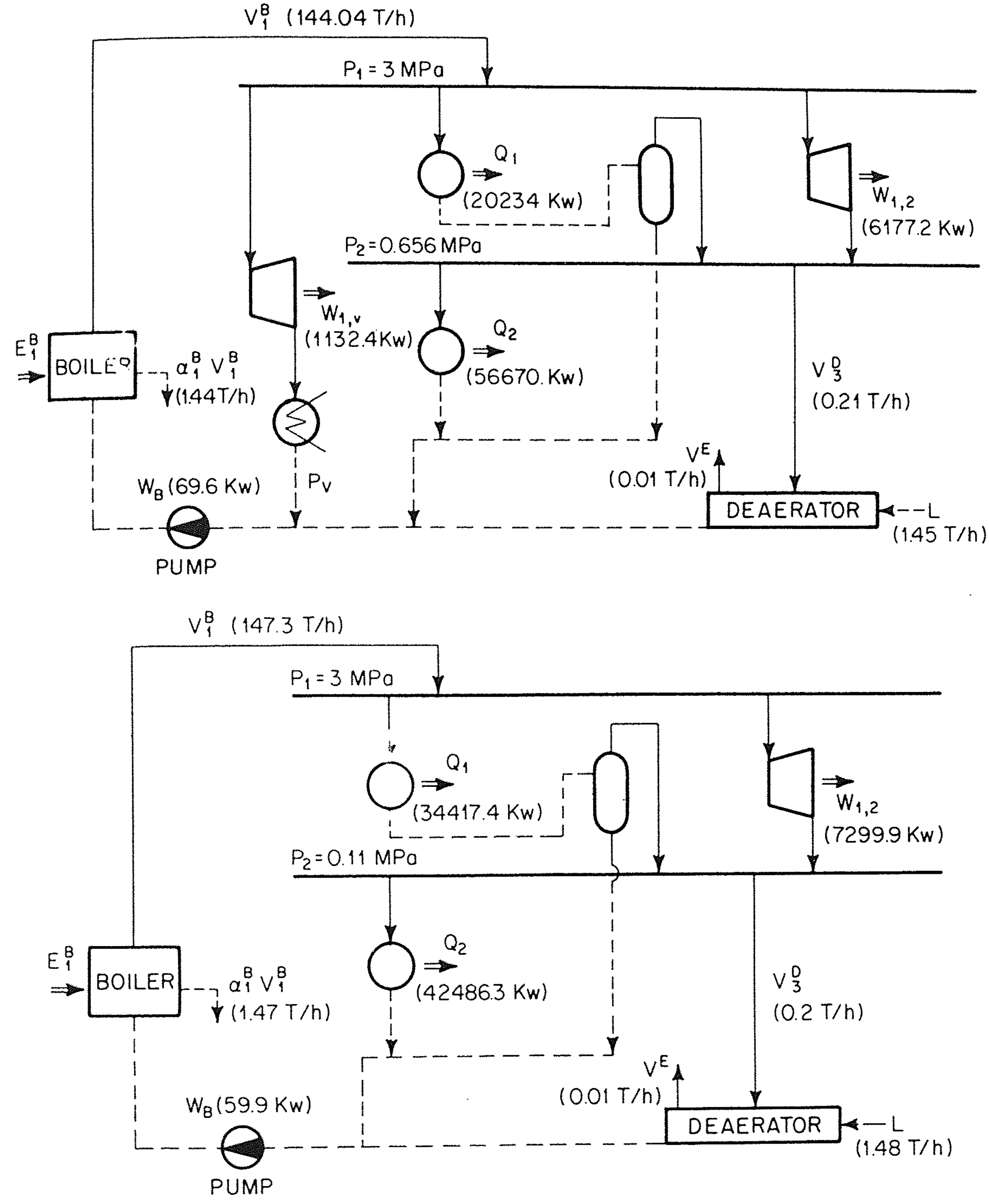

Fig. 8.– Optimal solutions. Example 1. 8*a*: N = 2; 8 *b*: N = 3.

satisfying process steam demand F_3 with steam coming from header 2 has a strong adverse effect in the system performance.

The advantage of exploring all these alternative and sub-optimal solutions to find structures that can satisfy restriction (27) where dedicated turbines are appropriately selected, is here transparent. It is very important to remember that the existence of these sub-optimal solutions is closely related to the fact that the system has achieved its maximum efficiency. Otherwise, the same sharp increase observed in Figure 7 for pressures P_3 below the optimum, occur above this value. Even though the variations observed in Figure 7 are of the same magnitude as those seen in the case of alternative optima, the name of sub-optimal solution was reserved for these structures to differentiate the origin of these changes. In the case of the alternative solutions the variations are originated in changes of turbine efficiencies, whereas in the case of sub-optimal structure the changes are also originated in variable heat and power demands.

IV. 8. *Interaction with the HEN synthesis problem*

When cycle efficiency is one, any increase in the HEN heat demand produces higher boiler energy load. Therefore, in this case of maximum efficiency, HEN minimum utility usage is a necessary optimal condition for the combined system. This coincides with the heuristic used in some HEN design algorithms. Other heuristic, which minimizes the number of units, is used to minimize capital costs. The SPS acting as an intermediary of the steam used for heating purposes may increase the minimum number of heating units. Consider any HEN designed assuming only one source of heating steam. Since the SPS has N headers, at the most N-1 additional units may arise for each of the specified heaters of the network. It is through this breakage that the minimum HEN heat load is mantained satisfying at the same time the power and other demands, without increasing the Main Boiler load. Note however that the existence of alternative and suboptimal solutions for the SPS can help in choosing a heat load distribution Q_j such that the number of heaters is minimized. This of course, does not always happen, but when it does, one may obtain a HEN design which satisfies minimum utility usage and minimum number of units. Note that the trade-off between area cost and energy costs was not considered above.

When the efficiency of the cycle is not maximum a trade off is established between the SPS and the HEN capital costs and the combined system operating cost. Additionally, in this case the SPS optimal solution includes condensing turbines and the heat load of these turbines (Q_V) can be used to meet HEN heat

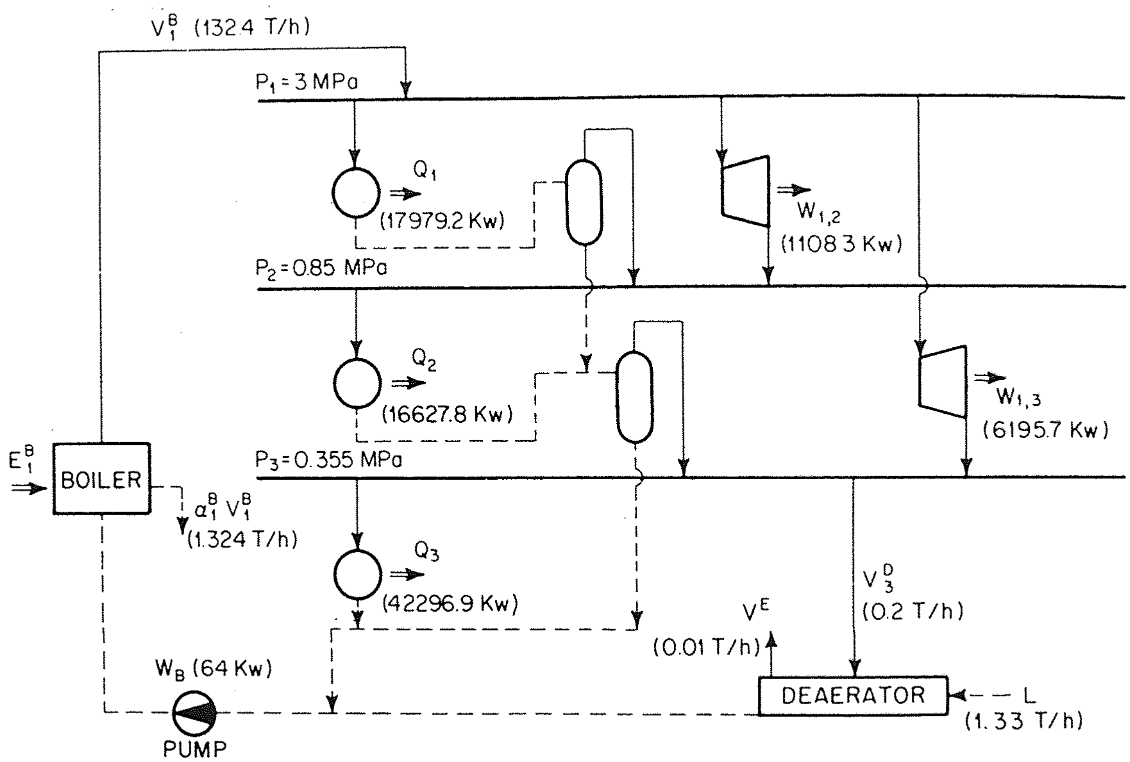

Fig. 9.– Optimal solution. Example 2.

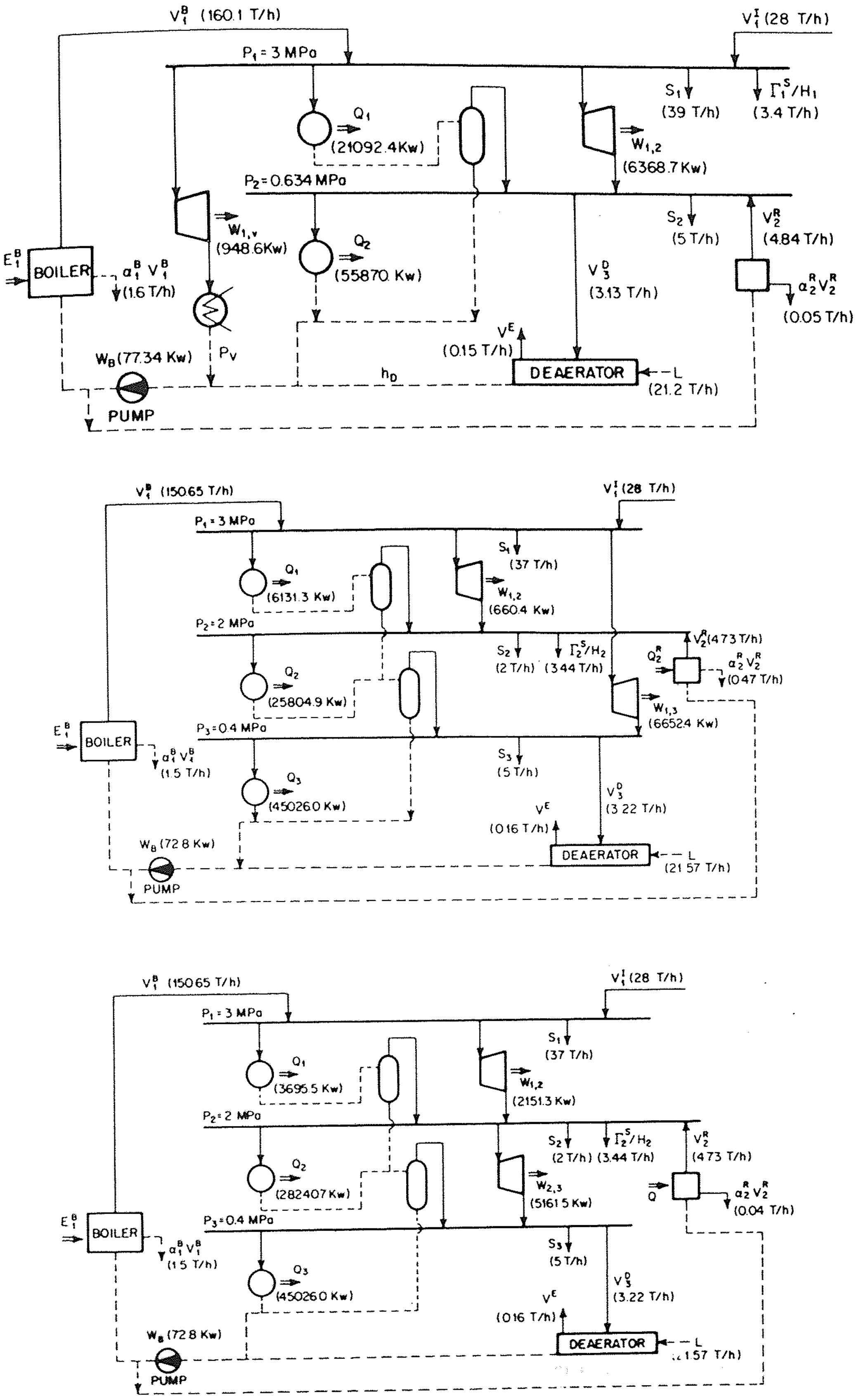

Fig. 10. - Optimal solutions. Example 3. 10*a*: N = 2; 10*b*: N = 3; 10*c*: N = 3 (alternative solution)

demands. Minimum HEN utility usage may not be necessary and the combined system can work with the same boiler load even if the HEN heat consumption is increased over the minimum. This relaxation of the HEN problem is very important and suggests further detailed studies. Similarly, when steam excesses in lower levels of pressures are present, a non-zero steam letdown $V_N^L \neq 0$ can exist. As in the case of Q_v the heat Q_N^L can be used to increase the HEN utility usage without increasing the boiler load. Note however that in both cases, if the temperature level of these heat excesses is below the pinch point of the HEN problem, increments of D_T only make cooling utility loads increase.

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Palabras clave

Sistemas de vapor y potencia - diseño óptimo - uso racional de energía - síntesis de procesos - diseño por computadora.

Nomenclature

a	binary variables
D	heat demand
d(T)	distribution of the heat demand
E	boiler energy consumption
F	demand of process steam (flow rate)
f	objective function of the MOCFP problem
H	vapor enthalpy
h	condensate enthalpy
L	flow rate of make up
M	number of demands
N	number of steam headers
P	pressure
Q	heat load
S	flow rate of process steam injection
T	temperature
V	steam flow rate
W	power

Greek letters

α	proportionality constant
ϵ	efficiency
Γ	energy flux demand as process steam
η	efficiency
Δ	temperature difference

Subscripts

B	boiler
D	dedicated turbine and deaerator
E	electrical power
i, j, k	headers i, j, k, respectively
L	water make-up
S	feed boiler water
V	condensing turbine
W	boiler feed water pump

Superscripts

B	boiler
D	dedicated turbine and deaerator
E	vapor leaving the deaerator
F	flash
G	process steam injection demand
H	heating steam
I	imported steam
L	letdown of steam
Min	minimum value
R	heat recovery boiler
(s)	saturation
S	process steam injection
$\underline{\underline{W}}$	turbine

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